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MODELING TWO-PHASE FLOWS WITH A PHASE INTERFACIAL SURFACE

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Many important problems for practice of the two-phase flow around bodies whose constituents are air (gas) and water (liquid) can be solved, exactly as in the case of a homogeneous medium, in a boundary-layer approximatoin that here retains the main structural criteria of a single-phase layer. However, depending on the phase mass relationships (on the degree of water content) important features appear whose crux is the formation and motion of a thin liquid layer over the streamlined surface.

A large number of papers is devoted to the study of stratified flows in order to simulate hydrodynamic processes being realized in different branches of engineering [1-5]. It should be noted that researchers turned the most attention mainly to the examination of two-phase flows with a laminar gas stream [2, 3] while the inhomogeneous structure of the phase separation boundary is not taken into account [4, 5] in the few papers devoted to two-phase flows with a turbulent boundary layer.

It is shown in [6, 7] that the flow of a liquid film subjected to an air stream in a sufficiently broad range of values of the air speed and water mass flow rate in the film is two-parametric in nature and depends on the air Reynolds number $Re_{x,2}$ and on the water film Re_1. It is found experimentally that the air—water phase interfacial surface is covered in all cases by a complex system of waves whose parameters are random in nature [6]. It is evident here that the air stream parameters influence the film motion while the nature of the liquid flow causes a change in the structure of the air medium.

The mathematical description of the mentioned phenomena is fraught with a number of difficulties including the complexity of taking account of all the processes proceeding in the film and the air stream that results in the necessity to introduce separate assumptions during execution of theoretical computations.

A flow model in the approximation of gas stream incompressibility in the absence of heat and mass transfer turns out to be sufficient for the examination of a number of physical processes. Such problems are encountered, say, in aviation engineering during determination of the aerodynamic characteristics of streamlined structures in the presence of thin liquid films.

A method and model of computing the combined flow of a water film with an air co-stream are proposed in this paper, which are based on the idea of merging the solutions of the air and liquid phase boundary layer equations. The conception of the model is configured in the representation of the laminar nature of the motion in the film and the turbulent nature in the air stream. The condition of continuity of the friction stress and velocity is posed at the boundary separating the two phases and its structure is assumed nonuniform. In the general case the flow is considered gradient and planar.

The air stream characteristics are analyzed by numerical integration of the system of differential equations

$$\bar{u}_2 \frac{\partial \bar{u}_2}{\partial x} + \bar{v}_2 \frac{\partial \bar{u}_2}{\partial y} = -\frac{1}{u_i}\frac{du_i}{dx} + \frac{d\bar{\tau}_2}{dy} - \bar{u}_2^2 \frac{1}{u_i}\frac{du_i}{dx}; \tag{1}$$

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$$\frac{\partial \bar{u}_2}{\partial x} + \frac{\partial \bar{v}_2}{\partial y} + \bar{u}_2 \frac{1}{u_i} \frac{\partial u_i}{\partial x} = 0, \tag{2}$$

where $\bar{u} = u/u_i$, $\bar{v} = v/u_i$, u_i is the velocity on the outer boundary of the shear layer; u and v are the longitudinal and tranverse components of the average velocity in the boundary layer; τ is the tangential stress; and x and y are longitudinal and transverse coordinates. Later, parameters of the water film are denoted by the subscript 1 in the formulas, and of the air stream by 2, while the abbreviation bo denotes parameters on the phase separation boundary.

The boundary conditions are the following

$$u_2 = u_{bo},\ v_2 = v_{bo},\ \tau_2 = \tau \qquad \text{for}\quad y = \delta_1,$$
$$u_2 \to u_i,\ \tau_2 \to 0 \qquad \text{for}\quad y = \delta_2.$$

Here δ_1 is the mean thickness of the water film. Since the second boundary condition is asymptotic, that value of the ordinate y for which $u_2 = 0.995u_i$ is taken as the thickness δ_2 constraining the computational domain.

The system is closed by using the Boussinesq formula

$$\tau_2 = \mu_e \partial u_2/\partial y,\ \mu_e = \mu_2 + \mu_{2T}$$

(μ_2, μ_{2T} are the dynamic coefficients of molecular and turbulent viscosity).

An algebraic model [8] is used to find μ_{2T}, which is well recommended in computations of a different class of gradient flows on both smooth and rough surfaces

$$\mu_{2T} = \rho_2 \chi \Delta v_* \gamma(\eta)\, \mathrm{th}\left(l\sqrt{\tau_*}/\chi\Delta\right),$$

where χ is an empirical constant; ℓ is the length of the mixing path; $\gamma(\eta)$ is a function taking account of the influence of flow alternation; $\eta = y\delta_2$ is the dimensionless transverse coordinate; $\Delta = \int_0^{\delta_2} (u_i^+ - u_2^+)dy$ is the Rott–Clauser length parameter; $u_2^+ = u_2/v_*$ is the lonitudinal component of the average velocity reduced to the scale of the wall law; $v_* = \sqrt{\tau_2}/\rho_2$ is the dynamic velocity, ρ_2 is the density; $\tau_* = 1 + \Phi\eta$ for $\Phi \geq 0$ and $\tau_* = 1/(1 - \Phi\eta)$ for $\Phi < 0$; $\Phi = (\delta_2/\tau_{bo})(dp/dx)$ is a pressure gradient parameter. Here

$$l = K(y + \Delta y)\,\mathrm{th}\,\frac{\mathrm{sh}^2\left(\chi_1 (y^+ + \Delta y^+)\right) \mathrm{th}\left(\mathrm{sh}^2\left(\chi_2 (y^+ + \Delta y^+)\right)\right)}{K(y^+ + \Delta y^+)\sqrt{\tau_*}},$$

(K, χ_1, χ_2 are empirical coefficients; $\Delta y^+ = \Delta y v_*/\nu_2$; Δy is a function that takes account of the influence of roughness). In conformity with [9]

$$\Delta y^+ = \begin{cases} \dfrac{1}{\chi_1} \mathrm{arth}\left[\chi_1 \left(\Delta u^+ + u_{bo}^+\right)\right] & \text{for}\quad h^+ \leqslant h^*, \\ h^+ \exp\left(K\left(B - u_{bo}^+\right)\right) & \text{for}\quad h^+ > h^*, \end{cases}$$
$$h^* = y^* \exp\left(K\left(B - u_{bo}^+\right)\right),\quad \Delta u^+ = (1/K)\ln h^+ - B + C_{sm}$$

Here Δu^+ is a function of the shift of the logarithmic section of the velocity profile due to the influence of roughness; B is a constant in the logarithmic law $u_2 = (1/K)\ln(y^+/h^+) +$ B and is determined in conformity with the results of experimental investigations [7] as $B = 75.4\mathrm{Re}_{x,2}^{-0.204}$; $h^+ = hv_*/\nu_2$; h is the height of the wave roughness for whose computation an approximate expression from [7] is used, and C_{sm} is the constant of the logarithmic law for a smooth surface.

The system (1) and (2) is solved by the method of lines. Values of the mentioned coefficients K and χ differ from the standards, by which a heterogeneous stream configuration is taken into account in a turbulent viscosity model. The magnitude of K is obtained in experiments [7] for which the law of the logarithmic velocity distribution was assumed valid over the whole thickness of the external stream. Since the Clauser constant χ is governing in the external domain in the μ_{2T} model used here, its magnitude for the heterogeneous stream χ_{ht} is found by converting the standard for a homogeneous stream value $\chi_{hs} = 0.0168$ and it is $\chi_{ht} = 0.0268$.

The derivation of the fundamental relationships to find the laminar liquid film parameters is represented in [10] where the final equations are presented, included in the total computation model in which the boundary conditions $u_1 = 0$, $v_1 = 0$ for $y = 0$, $u_1 = u_2 = u_{bo}$, $\tau_1 = \tau_2 = \tau_{bo}$ for $y = \delta_1$ are taken into account.

The nature of the velocity distribution in a liquid film moving under the effect of an air stream can be estimated from the expression

$$u_1 = \frac{\tau_{bo}}{\mu_1}\left[y - \rho_2 u_i \frac{du_i}{dx} \frac{y}{\tau_{bo}}\left(\frac{y}{2} - \delta_1\right)\right], \tag{3}$$

where y is the transverse coordinate that permits obtaining the magnitude of the velocity on the air–water interfacial surface $u_{bo} = (\tau_{bo}/\mu_1)\cdot\delta_1 + (1/2)\rho_2 u_i(du_i/dx)\cdot(\delta_1{}^2/\mu_1)$ upon substitution of the value of the film thickness δ_1. The mean water film thickness is

$$\delta_1 = \left[2G_1\nu_1\left(\frac{1}{\tau_{bo} + \frac{2}{3}\rho_2 u_i \frac{du_i}{dx}\delta_1}\right)\right]^{0,5} \tag{4}$$

(ν_1 is the kinematic viscosity, and B_1 is the liquid mass flow rate in the film).

Computation of the flow starts with giving the initial velocity profile. To this end, the ordinary differential equation obtained from (1) and (2) by the Blasius transformation ($\eta = u_i y/\sqrt{\nu_2 x}$, $\bar{u}_2 = \partial\varphi/\partial\eta$):

$$\varphi''' + \frac{m+1}{2}\varphi\varphi'' + m\left[1 - (\varphi')^2\right] + \varphi''\frac{1}{\mu_2}\frac{\partial\mu_{2T}}{\partial\eta} = 0 \tag{5}$$

is integrated (φ is the stream function and $m = (x_0/u_i)(\partial u_i/\partial x)$ is the pressure gradient parameter while x_0 is the coordinate of the initial values).

The results of computations as well as data of [11] permit making a deduction about the sufficiently rapid localization of errors associated with inaccuracy in giving the initial conditions on the basis of (5).

The boundary conditions become the following

$$\varphi = 0,\ \varphi' = u_{bo} \text{ for } y = \delta_1,$$
$$\varphi' = 1,\ \varphi'' = 0 \text{ for } y = \delta_2.$$

Equation (5), reduced first to a system of three first-order ordinary differential equations is integrated by the Runge–Kutta fourth-order method. The unknown value of φ_0 for $y = 0$ is determined by the shooting method by confirming satisfaction of the boundary conditions on the upper boundary. Each approximation v_* is used here to determine the drag coefficient c_f by which the values of the film thickness and the velocity on the phase interfacial boundary needed to give the boundary condition on the lower boundary are computed in conformity with (3) and (4). An analogous merging procedure is performed for the solutions in each succeeding step in the coordinate x.

Systematic computations of two-phase flows with a phase separation surface were performed as a result of a programmed realization of the considerations elucidated above. Taken as the parameter by which the comparison between the computations and the results of the experimental investigations was realized was δ_1. To illustrate the possibilities

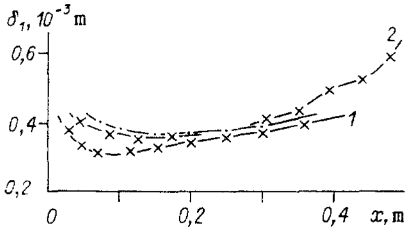

Fig. 1

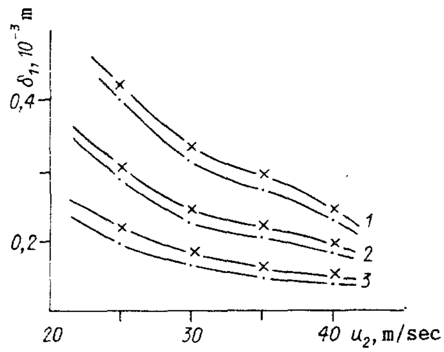

Fig. 2

of the computation method and model proposed, conditions of experiments of a number of authors [6, 12] were modeled numerically with subsequent comparison of the computed and experimental data.

The change in film thickness δ_1 along the length of a flat plate is shown in Fig. 1, and from which a judgment can be made about the nature of the flow of a thin water layer subjected to an air stream [the points display the experimental values [12], and the crosses the calculated values; curves 1) $dp/dx = 0$; 2) $dp/dx \neq 0$, $u_2 = 20$ m/sec, $Re_1 = 198$]. The comparisons between computations and experiments [6] represented in Fig. 2 (curves 1-3 and $Re_1 = 400, 200, 100$) are most indicative in the plan of estimating the reliability of theoretical investigations performed using different air and liquid film Reynolds numbers.

Results of computations of gradient flows being realized on the upper surface of a NACA-2211 wing profile at zero angle of attack (see Fig. 1, curve 2) indicate a change in the nature of the film flow and also permit making a deduction about the magnification of the action of the pressure gradient in the presence of a thin water layer on the surface of the streamlined body, which can result in premature separation of the air stream under individual conditions.

However, the computations of δ_1 presented for values of dp/dz different from zero require additional experimental verification and refinement of the parameters governing the wave roughness of the liquid film surface.

As follows from an analysis of Figs. 1 and 2, despite the qualitative correspondence achieved, a systematic deviation of the computed film thickness as compared with the measured values holds in all cases. Such a qualitative discrepancy can be explained by the number of assumptions taken earlier and, in particular, by not taking account of the turbulizing action of the external factors on the nature of the water film flow.

Meanwhile the results obtained indicated the possibility of applying this approach to modeling and computing this kind of heterogeneous flows.

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